Solutions to random exercises from Chapter 2

2.1

2. Slope is 0.5.

6. $\dfrac{9-0}{0--3}=\dfrac{9}{3}=3$

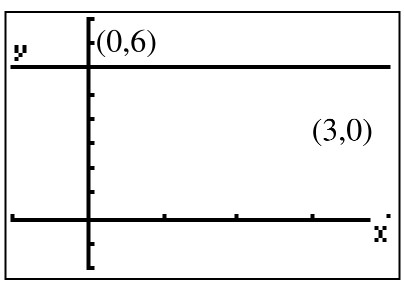
11. $\dfrac{\$5}{1\text{ year}}$. Over the next decade, the annual fees are to increase by \$5 per year.

12. $\dfrac{\$12,800-\$8,200}{8\text{ years}}=\dfrac{\$4,600}{8\text{ years}}=\dfrac{\$575}{1\text{ year}}$. Over the past 8 years the car has decreased in value an average of \$575 per year.

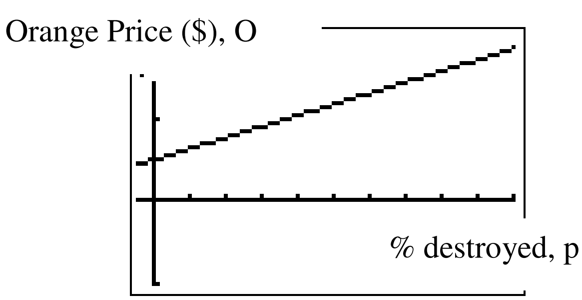
15. $\dfrac{\$0}{1\text{ year}}$. Over the next 3 years, the salary will not change.

16. $\dfrac{\$10-\$7}{6\text{ 6-month periods}}=\dfrac{\$3}{6\text{ 6-month periods}}=\dfrac{\$0.5}{\text{6-month period}}$. For every 6-month period over the next 3 years, the retail workers will receive a \$0.50 raise.

20.



27.

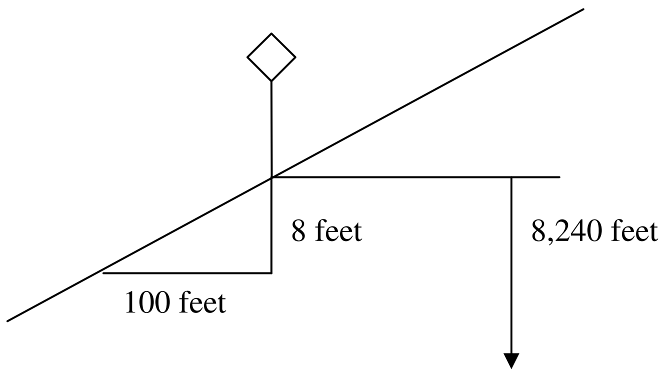


$$\frac{\$1.50-\$0.50}{75\%}=\frac{\$1.00}{75\%};\ O(p)=0.50+\frac{1}{75}p$$

28. If we let t = 0 be 1980, then $\dfrac{232.1-412.1}{23-0}=\dfrac{-7.8 \text{ deaths per } 100{,}000}{1 \text{ year}}$. We can model the death rate due to heart disease as $D(t)=412.1-7.8t$.

29. If we let t = 0 be 2000, then $D(t)=0.215-0.01t$.

30.



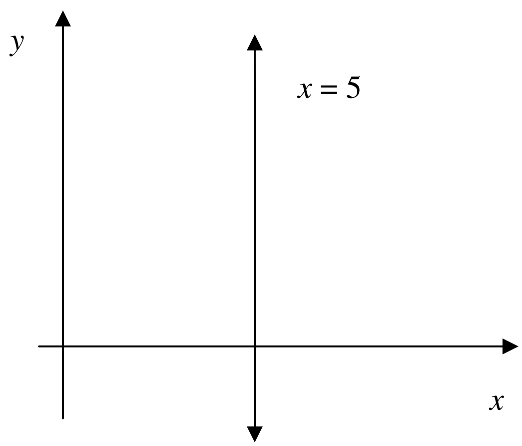
If we let R = road elevation above sea level in feet and d = horizontal distance from the sign, we have $R(d)=8240-\dfrac{8}{100}d$

33. A horizontal line has 0 slope because as x changes, y does not or $\dfrac{\Delta y}{\Delta x}=\dfrac{0}{\Delta x}=0$. A vertical line has undefined slope because x will not change and y will or $\dfrac{\Delta y}{\Delta x}=\dfrac{\Delta y}{0}=\text{undefined}$.

34. In the slope formula $\dfrac{f(b)-f(a)}{b-a}$, the numerator $f(b)-f(a)$ represents the vertical change (rise) and the denominator $b-a$ represents the horizontal change (run).

35. Because as the magnitude of the slope increases, the vertical change becomes greater for the same horizontal change.

37. Yes. We can see graphically that the only line that does not have a vertical intercept is a vertical line which is not a **function**. For instance,



38. No slope and undefined slope both refer to the slope of vertical lines. Zero slope refers to the slope of horizontal lines.

48. If we let S = speed in mph and t = time in seconds, $S(t) = 50 + 1t$.

55. If we let B = loan balance in \$ and m = number of monthly payments, $B(m) = 2970 - 135m$.

64. To be linear there must be a constant rate of change therefore we must find the rate of change. $\frac{19-13}{8-5} = \frac{6}{3} = 2$. Therefore, $\begin{array}{l} b = 19 + 2(2) \\ b = 23 \end{array}$.

2.2

1.

$$\begin{array}{l} y - 10 = -4x \\ \frac{y-10}{-4} = x \end{array} \quad ; \; f^{-1}(y) = \frac{y-10}{-4}$$

4.

$$\begin{array}{l} y + 15 = 6x \\ \frac{y+15}{6} = x \end{array} \quad ; \; f^{-1}(y) = \frac{y+15}{6}$$

6. Linear because the cost per gallon is constant. $C(g) = 1.889g$ where C = the total cost of gasoline purchased and g = number of gallons purchased.

7. Linear because the cost per gallon is constant. $C(g) = 2.339g$ where C = the total cost of gasoline purchased and g = number of gallons purchased.

8. Linear because the cost per pair of pants will be constant. We first find the cost per pair of pants when the cost is \$103.50 for a six pack $\frac{103.5}{6} = 17.25$. With a 100% markup (double the cost) 17.25(2) = 34.50. If we let R = total retail revenue and p = number of pairs of pants, we get the function $R(p) = 34.5p$.

9. Linear because each top sold will bring in a constant revenue of \$93. $R(t) = 93t$ where R = the total retail revenue and t = number of tops sold.

10. Not linear. The change is not constant. (Note: This data set is nearly linear and could be approximated with a linear function. See Section 2.3 for details on how this is done.)

11. The total revenue (in millions of \$) for McDonald's, t, as a function of the fiscal year, y, is near linear because the rate of change between consecutive years is nearly constant. (answers vary) $T(y) = 14243 + 1395y$.

12. We cannot tell if these data are linear or near linear given only two data points.

13. We cannot tell if these data are linear or near linear given only two data points.

14. The number of children in Madagascar (in thousands), C, as a function of the years since 1990, t, is near linear because the rate of change between consecutive years is nearly constant. (answers vary) $C(t) = 2120 + 76t$.

15. These data are not linear or near linear because the change in per capita spending on prescription drugs per year varies too much and is not nearly constant.

16. "constant rate of change", "directly proportional", "increased/decreased by the same amount", etc.

22 a. $l(t) = 162 + \frac{17}{300}t$; $\frac{17 \text{ inches}}{300 \text{ years}}$

b. $$\begin{aligned} l(99) &= 162 + \frac{17}{300}(99) \\ &= 167.61'' \end{aligned}$$

$$\begin{aligned} 101 &= 14.08p - 53.87 \\ 154 &= 14.08p \\ 10.99 &= p \end{aligned}$$

23. When the death rate is 101 per 100,000 the percentage of smokers is about 11%.

$$\begin{aligned} 61.4 &= -2.606t + 131.8 \\ -70.4 &= -2.606t \\ 27.01 &= t \end{aligned}$$

24. ; In 27 days after the end of May 2006 (June 27, 2006) the snow runoff volume is 61 thousand acre feet.

$$\begin{aligned} 251.5 &= 10.47t - 146.8 \\ 10 &= t \end{aligned}$$

25. When the marketing labor cost is 251.5 billion dollars it is the year 2000.

26. It depends if the function is truly linear for all values (including for values not provided). If so, then a linear function will perfectly forecast unknown values.

27. Function modeling is valuable when we have enough data provided to make reasonable assumptions about patterns/trends. If we get an accurate model we must use caution in terms of how far into the future we predict (extrapolation) due to potential changes.

28. I disagree. $f^{-1}(x)$ is inverse notation and represents a function in which the input and output values have been switched. $\frac{1}{f(x)}$ is the reciprocal of all output values. For instance, if $f(x) = 4$, $\frac{1}{f(x)} = \frac{1}{4}$.

2.3

1. $y_1 = \frac{7}{4}x + 1$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
0	1	1	0	0
2	5	4.5	0.5	0.25
4	6	8	-2	4
6	12	11.5	0.5	0.25
8	16	15	1	1
10	19	18.5	0.5	0.25
12	20	22	-2	4
14	25	25.5	0.5	0.25
16	29	29	0	0
				Sum of squares: 10

$y_2 = \frac{3}{2}x + 2$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
0	1	2	-1	1
2	5	5	0	0
4	6	8	-2	4
6	12	11	1	1
8	16	14	2	4
10	19	17	2	4
12	20	20	0	0
14	25	23	2	4
16	29	26	3	9
				Sum of squares: 27

$y_1 = \frac{7}{4}x + 1$ fits the data better because the sum of squares are smaller indicating that there is less error with this model.

2. $y_1 = \frac{-5}{2}x + 10$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
0	10	10	0	0
1	8	7.5	0.5	0.25
2	4	5	-1	1
3	3	2.5	0.5	0.25
4	0	0	0	0
				Sum of squares: 1.5

$y_2 = \frac{-7}{3}x + 10$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
0	10	10	0	0
1	8	7.67	0.33	0.1089
2	4	5.3	-1.3	1.69
3	3	3	0	0
4	0	0.67	-0.67	0.4489
				Sum of squares: 2.2478

$y_1 = \frac{-5}{2}x + 10$ fits the data better because the sum of squares are smaller indicating that there is less error with this model.

3.

$y_1 = 17.5x + 12.5$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
5	100	100	0	0
10	125	187.5	-62.5	3906.25
15	320	275	45	2025
20	430	362.5	67.5	4556.25
25	450	450	0	0
				Sum of squares: 10,487.5

$y_2 = 30.5x - 180$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
5	100	-27.5	127.5	16256.25
10	125	125	0	0
15	320	277.5	42.5	1806.25
20	430	430	0	0
25	450	582.5	-132.5	17556.25
				Sum of squares: 35,618.75

$y_1 = 17.5x + 12.5$ fits the data better because the sum of squares are smaller indicating that there is less error with this model.

4.

$y_1 = \frac{13}{8}x + \frac{75}{4}$

x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
10	35	35	0	0
20	40	51.25	-11.25	126.5625
30	75	67.5	7.5	56.25
40	70	83.75	-13.75	189.0625
50	100	100	0	0
				Sum of squares: 371.875

$y_2 = 2x$

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$
10	35	20	15	225
20	40	40	0	0
30	75	60	15	225
40	70	80	-10	100
50	100	100	0	0
				Sum of squares: 550

$y_1 = \frac{13}{8}x + \frac{75}{4}$ fits the data better because the sum of squares are smaller indicating that there is less error with this model.

5.

$y_1 = -3x + 15$

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$
0	15	15	0	0
1	10	12	-2	4
2	9	9	0	0
3	4	6	-2	4
4	2	3	-1	1
				Sum of squares: 9

$y_2 = -3x + 13.9$

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$
0	15	13.9	1.1	1.21
1	10	10.9	-0.9	0.81
2	9	7.9	1.1	1.21
3	4	4.9	-0.9	0.81
4	2	1.9	0.1	0.81
				Sum of squares: 4.05

$y_2 = -3x + 13.9$ fits the data better because the sum of squares are smaller indicating that there is less error with this model.

6a. $T(I) = -2149.48 + 188.19I$

b. The slope means that for every increase in \$1000 in taxable income an estimated additional \$188.19 in taxes is due. The vertical intercept means that

when a married couple has an income of \$0 they owe -\$2149.48 in taxes. This is unreasonable so they either owe \$0 in taxes or may receive a refund.

c.

$$\begin{aligned} T(72) &= -2149.48 + 188.19(72) \\ &= \$11,400.24 \end{aligned}$$ This is interpolation.

d.

$$\begin{aligned} T(110) &= -2149.48 + 188.19(110) \\ &= \$18,551.48 \end{aligned}$$ This is extrapolation.

7a. $H(t) = 76.6 + 1.25t$

b. The slope means that the number of households increases by 1.25 million each year. The vertical intercept means that in 1980 the estimated number of households with TVs is 76,600,000.

c.

$$\begin{aligned} H(5) &= 76.6 + 1.25(5) \\ &= 82.85 \text{ million} \end{aligned}$$ This is interpolation.

d.

$$\begin{aligned} H(30) &= 76.6 + 1.25(30) \\ &= 114.1 \text{ million} \end{aligned}$$ This is extrapolation.

e. With a coefficient of determination of $r^2 = 0.97$ we would think they are accurate. We would be less sure of extrapolating because of possible future changes.

8a. $P(t) = -0.248t + 5.986$

b. The slope means that the carbon monoxide pollutant concentration is decreasing 0.248 parts per million each year on average. The vertical intercept means that in the year 0 (1990) the carbon monoxide pollutant concentration is 5.986 parts per million.

c.

$$\begin{aligned} P(2) &= -0.248(2) + 5.986 \\ &= 5.49 \end{aligned}$$ This is interpolation.

d.

$$\begin{aligned} P(10) &= -0.248(10) + 5.986 \\ &= 3.506 \end{aligned}$$ This is interpolation.

e. The coefficient of determination is $r^2 = 0.9949$. This gives us the indication that any predictions are pretty accurate.

9a. $H(g) = 2.39 + 0.05g$

b. The slope means that Luis Gonzales hit 0.05 home runs each game. The vertical intercept means that at game 0 Luis Gonzales had hit 2.39 homeruns which is of course unreasonable.

c.

$$\begin{aligned} H(100) &= 2.39 + 0.05(100) \\ &= 7.39 \end{aligned}$$ This is extrapolation.

d.

$$\begin{aligned} H(162) &= 2.39 + 0.05(162) \\ &= 10.49 \end{aligned}$$ This is extrapolation.

e. The coefficient of determination is $r^2 = 0.75$. This gives us the indication that any predictions would not be very accurate.

10a. $H(n) = 838.72 + 5.99n$

b. The slope means that for each story added to a building it increases the height by 5.99 feet. The vertical intercept means that a building with 0 stories the building would be 838.72' which is of course unreasonable.

c.

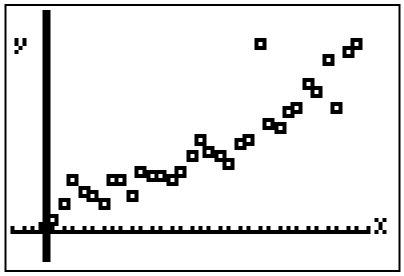
$$\begin{aligned} H(50) &= 838.72 + 5.99(50) \\ &= 1138.22' \end{aligned}$$ This is extrapolation.

d.

$$\begin{aligned} H(120) &= 838.72 + 5.99(120) \\ &= 1557.52' \end{aligned}$$ This is extrapolation.

e. The coefficient of determination is $r^2 = 0.30$. This gives us the indication that any predictions would not be very accurate.

11a.

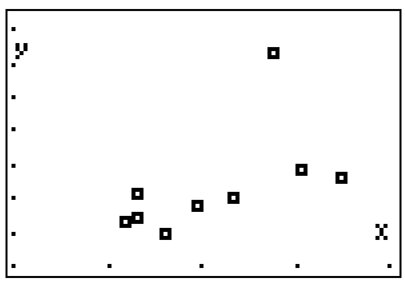


b. $M(y) = 14.55 + 10.72y$

c. The coefficient of determination is $r^2 \approx 0.84$ and the correlation is $r \approx 0.92$. This gives us the indication that any predictions would be reasonably accurate.

d.
$$450 = 14.55 + 10.72y$$
$$435.45 = 10.72y$$
$$\frac{435.45}{10.72} = y$$
$$41 \approx y$$

12a.



b. $M(b) = -599.71 + 1.08b$

c. The slope means that for every additional 1000 registered boats there are 1.08 more manatee deaths. The vertical intercept means that when there are 0 registered boats there are -599.71 manatee deaths which is, of course, unreasonable.

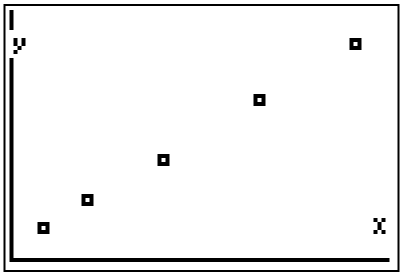
d. The coefficient of determination is $r^2 = 0.32$. This gives us the indication that any predictions would not be very accurate.

e.

$$\begin{aligned} M(3000) &= -599.71 + 1.08(3000) \\ &= 2640.29 \end{aligned}$$

Even though we get a numerical answer (2640.29) for the number of manatee deaths this model should not be used to extrapolate so far.

13a.



b. $L(f) = 7865.13 + 0.84f$ According to the model $L(f)$ when the food, beverage, and packaging costs are $0 the labor costs would be $7,865,130. Also, for each $1000 increase in food, beverage, and packaging costs the labor costs increase by $840.

c. Since $r^2 = 0.999$ and $r = 0.999$ the model fits the data well and the predictions may be quite accurate.

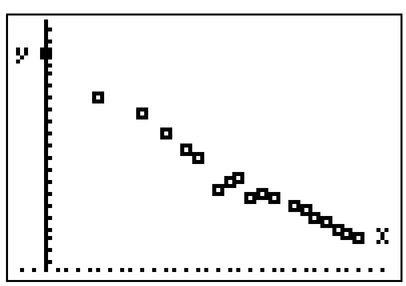
d.

$$\begin{aligned} 200{,}000 &= 7865.13 + 0.84f \\ 192134.87 &= 0.84f \\ 228731.98 &= f \end{aligned}$$

$$\$228{,}731{,}980$$

This extrapolates too far out.

14a.



b. $P(t) = 35.71 - 0.52t$

c. Since $r^2 = 0.97$ and $r = -0.986$ the model fits the data well and the predictions may be quite accurate.

d.

$$\begin{aligned} 0 &= 35.71 - 0.52t \\ -35.71 &= -0.52t \\ \frac{-35.71}{-0.52} &= t \\ 68.67 &\approx t \end{aligned}$$

This would be year 2043 but this would be extrapolating too far out and it is very unlikely that there will ever be no people smoking.

16. It is a method of selecting the best linear function to model data and generally used to make predictions.

17. The coefficient of determination is a way of determining the percentage of data values that are used to calculate the line of best fit. Therefore, the closer the coefficient of determination is to 1 (or 100%) the more accurate the line of best fit will be.

18. Interpolation is the process of obtaining a value from a graph or table that is located between major points given, or between data points plotted. Extrapolation is the process of obtaining a value from a chart or graph that extends beyond the given data. The "trend" of the data is extended past the last point given or before the first point and an estimate is made of the value.

19. Both values help us to determine the accuracy of the model in making predictions through interpolation and extrapolation.